BACKGROUND

Reconciling Vectors

Exercise 12.1 Adding Vectors

Objective

At the conclusion of this exercise, you will be able to do the following:

1. Reconcile vectors using the parallelogram method of vector addition
2. Reconcile vectors using the tip-to-tail method of vector addition
3. Reconcile vectors using the component method of vector addition

Procedure

Read the section on vector addition (pp. 403–406) and the section on free-body diagrams (p. 409) in Chapter 12, "Statics," of your *Principles of Engineering* textbook.

Materials

- Protractor
- Standard inch rule, with an accuracy of 1/8 inch (see Figure 14-4 on page 449 of your textbook)

Problem 12.1 Figure 12-1 is a scaled vector drawing representing two forces, F_1 and F_2, that act on an object.

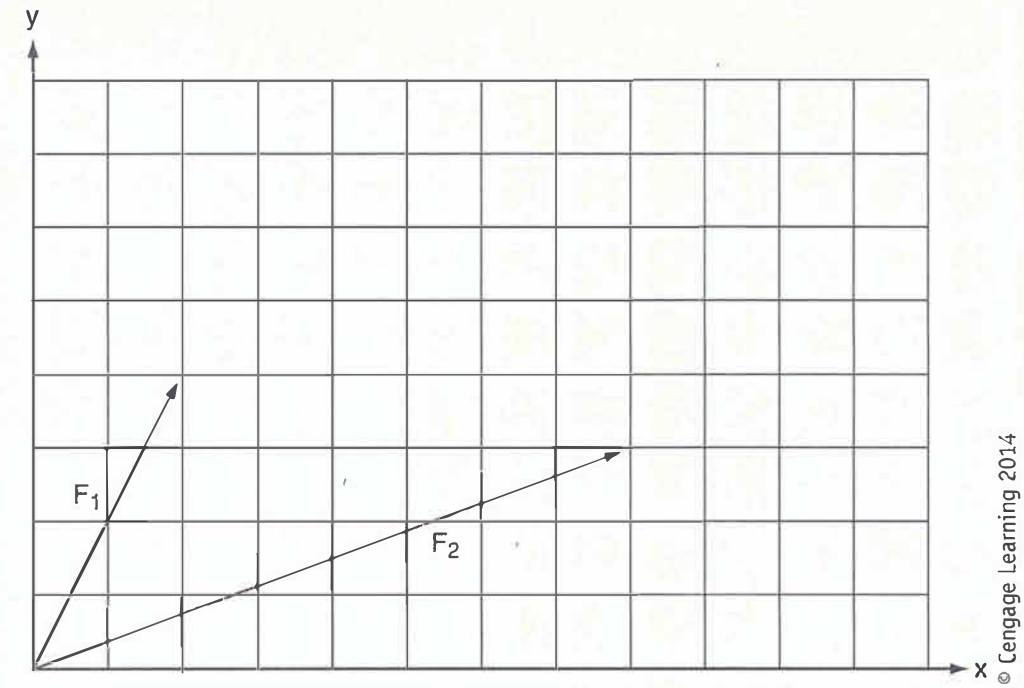

FIGURE 12-1 *Vector addition—parallelogram method.*

1. Draw the resultant force vector (F_R) on Figure 12-1 using the *parallelogram method* of vector addition, described on page 404 of your textbook.
2. Measure the length of resultant force vector (FR) to the nearest 1/8-inch and record its length in the space provided.
   Length of F_R = ____________in.
3. Determine the magnitude of the resultant force (F_R) using the scale (1/8 in. = 1 lb) and record the value in the space provided.
   F_R = ____________lb
4. Measure the angle of resultant force vector (F_R) relative to the horizontal axis and record its direction in the space provided.
   θ = ____________°

Problem 12.2 Figure 12-2 is a scaled vector drawing representing two forces, F_1 and F_2, that act on an object.

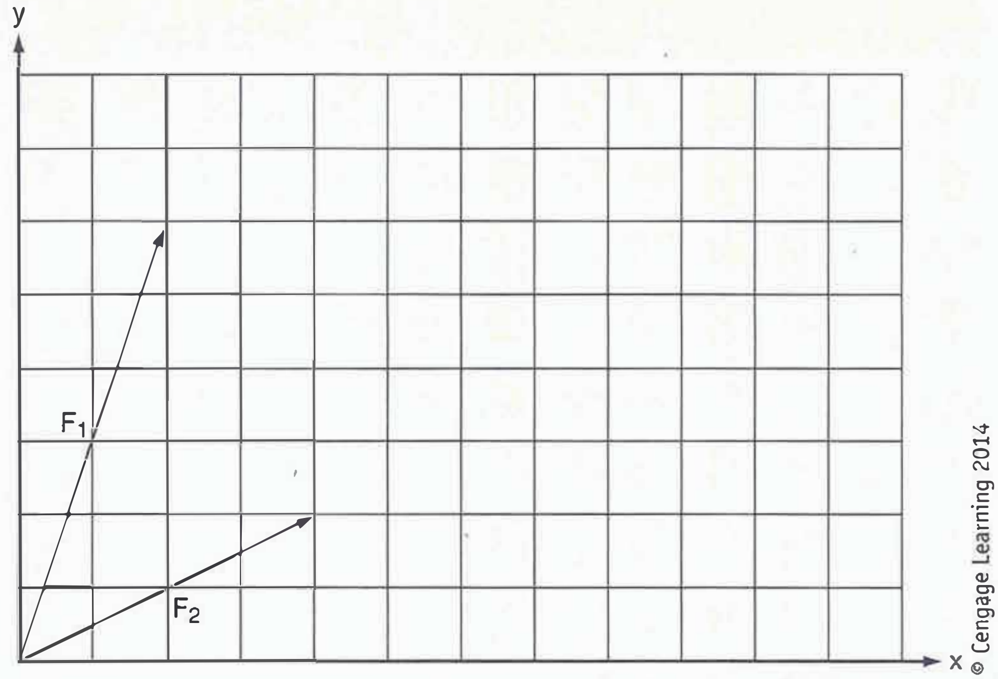

FIGURE 12-2 *Vector addition—parallelogram method.*

1. Draw the resultant force vector (FR) on Figure 12-2 using the *parallelogram method* of vector addition described on page 404 of your textbook.
2. Measure the length of resultant force vector (F_R) to the nearest 1/8-inch and record its length in the space provided.
   Length of F_R = ______________in.
3. Determine the magnitude of the resultant force (F_R) using the scale 1/8 in. = 1 lb and record the value in the space provided.
   F_R = ______________lb
4. Measure the angle of resultant force vector (FR) relative to the horizontal axis and record its direction in the space provided.
   θ = ______________°

Problem 12.3 Figure 12-3a is a scaled vector drawing representing three forces, F_1, F_2, and F_3, that all act on an object.

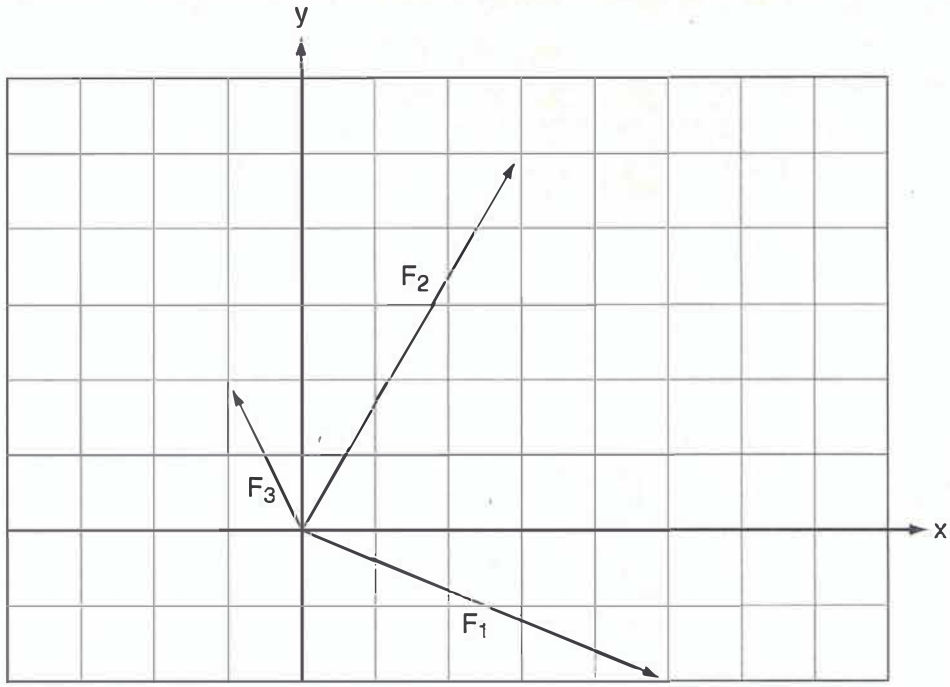

FIGURE 12-3a *Vector addition—tip-to-tail method.*

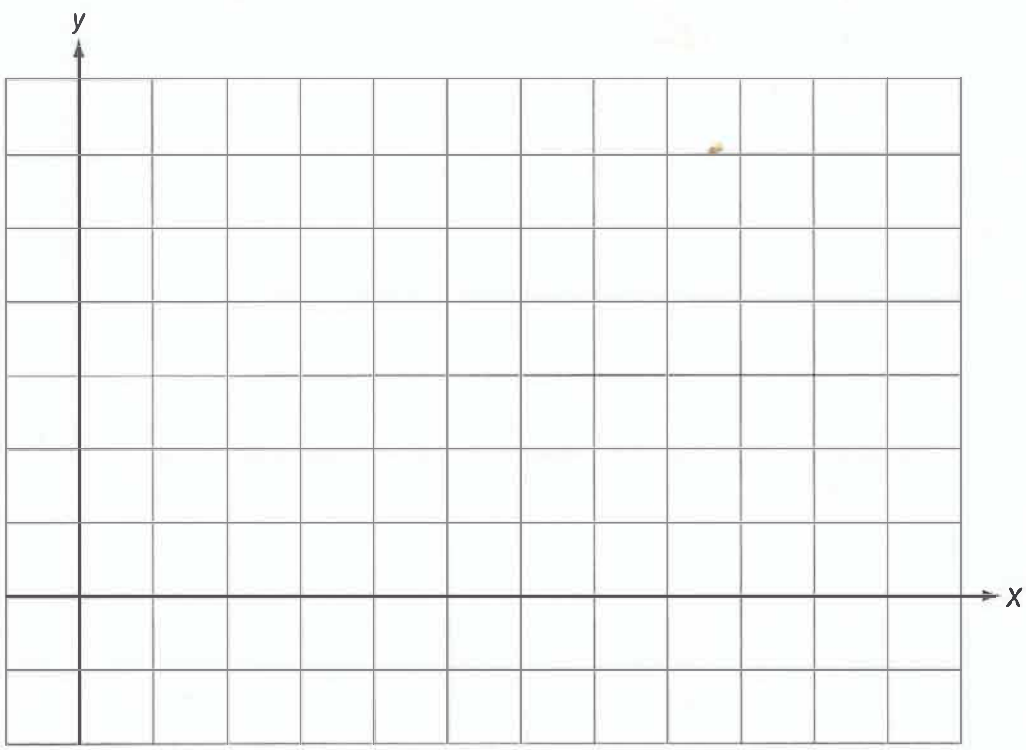

FIGURE 12-3b *Using the tip-to-tail method to draw vectors.*

1. Redraw force vectors F_1, F_2, and F_3 on Figure 12-3b using the *tip-to-tail* method of vector addition described in your textbook. Then draw the resultant force vector (F_R) to complete the exercise.
2. Measure the length of resultant force vector (F_R) on Figure 12-3b to the nearest 1/8-inch and record its length in the space provided.
   Length of F_R = ______________in.

3. Determine the magnitude of the resultant force (F_R) using the scale 1/8 in. = 1 lb and record the value in the space provided.
   F_R = ____________lb
4. Measure the angle of resultant force vector (F_R) on Figure 12-3a relative to the horizontal axis and record its direction in the space provided.
   θ = ____________°

TIP SHEET

A vector that is directed at an angle less than 90° with respect to the *x*-axis or *y*-axis can be resolved into its horizontal (*x*) and vertical (*y*) components. This is true for all vector quantities (force, displacement, velocity, acceleration, etc.). For example, if a sled is pulled by a rope at an angle that is 40° above the horizontal (*x*) axis (Figure 12-4), the rope is pulled (or has tension) both upward and to the right. Therefore, the tension in the rope, represented by the *force vector* (F), is made up of two components: one that represents the vertical component of the force (F_Y) and one that represents the horizontal component of the force (F_X), as illustrated in Figure 12-4.

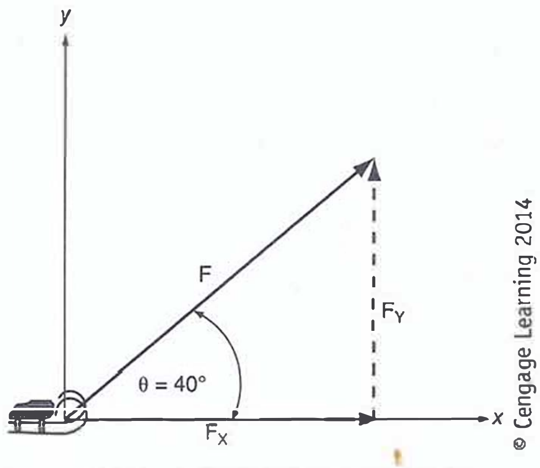

FIGURE 12-4 *The vertical and horizontal components of force vector F.*

When a vector is represented graphically, with its proportional vertical and horizontal component vectors (F_X and F_Y) drawn tip-to-tail, the three vectors form a right triangle (as in Figure 12-4). The magnitudes of F_X and F_Y can be calculated using the following trigonometric equations: $F_x = F\cos\theta$, and $F_y = F\sin\theta$. The resultant force vector (F) can be calculated using the Pythagorean theorem ($F^2 = F_x^2 + F_y^2$) if the magnitude of both the horizontal (F_X) and vertical (F_Y) component vectors is known.

Problem 12.4 Figure 12-5 shows the vector representation of resultant force (F) and its vertical (F_Y) and horizontal (F_X) component vectors.

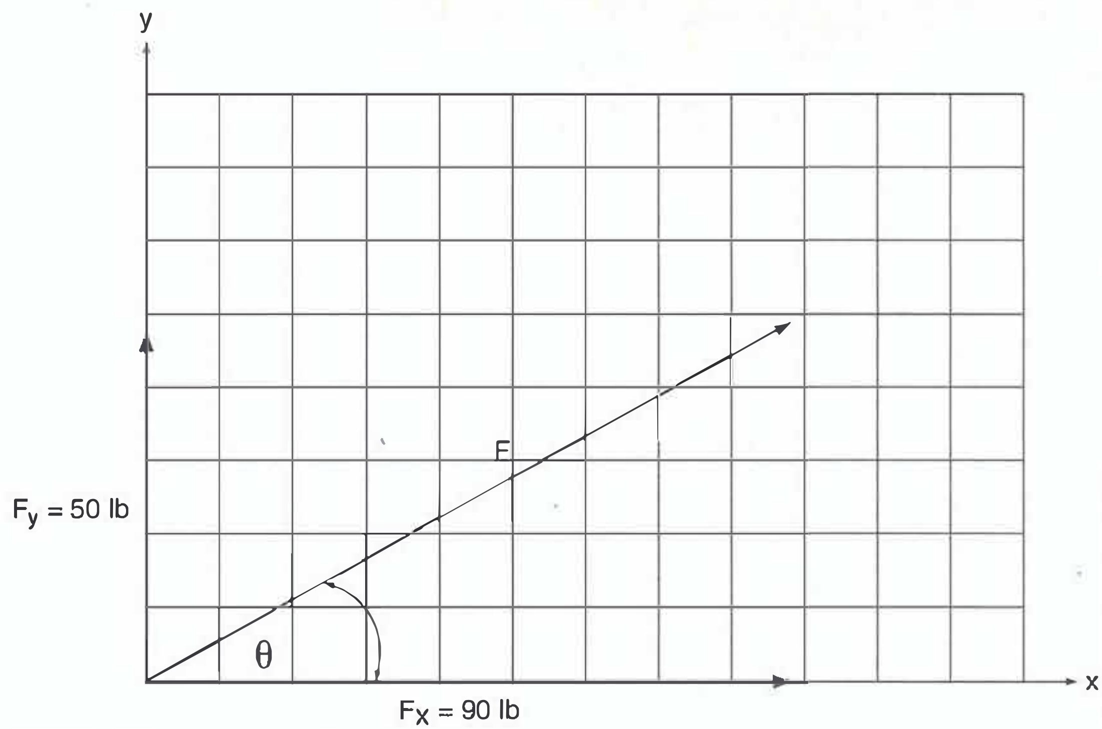

FIGURE 12-5 *Vector resolution—component method.*

1. Redraw F_y so that its tail begins at the tip of F_x and its tip touches the tip of F. Label this force F_y.
2. Calculate the magnitude of force (F) shown in Figure 12-4 using the Pythagorean theorem. Show your math work and record your answer in the space provided.

F = ________________lb

3. Calculate the angle (θ) of force vector (F). Show your math work and record your answer in the space provided.

θ = ______________________

Problem 12.5 Figure 12-6 is the vector representation of a 120-lb force (F) exerted at an angle of 50° relative to the horizontal axis and the horizontal (F_X) and vertical (F_Y) components of force F.

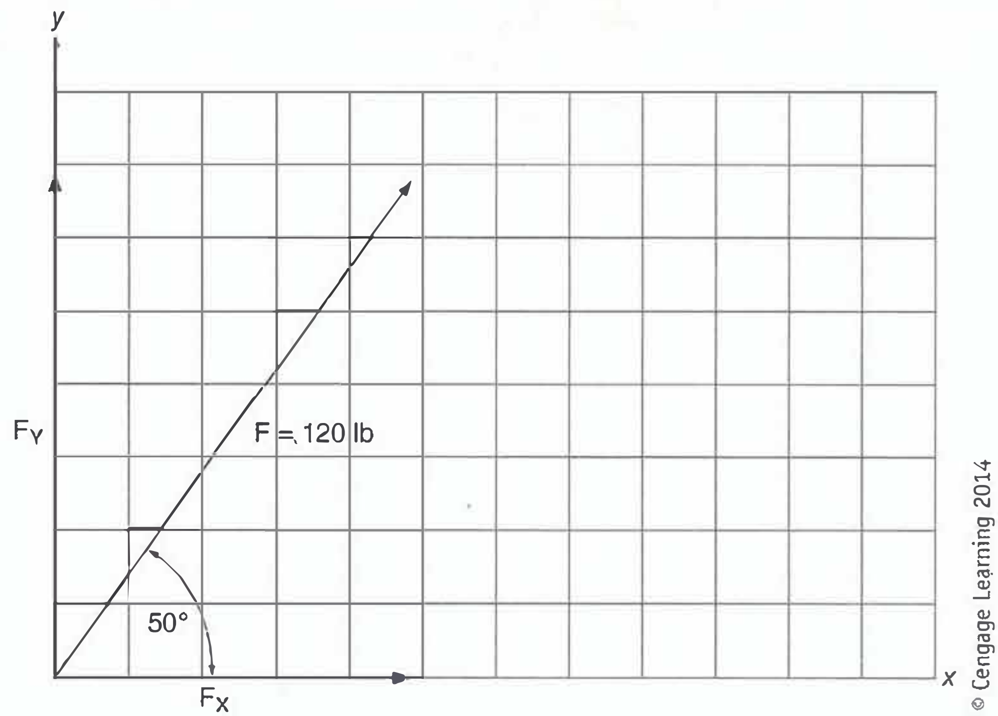

FIGURE 12-6 *Vector resolution—component method.*

1. Redraw F_y so that its tail begins at the tip of F_x and its tip touches the tip of F. Label this force F_y.
2. Calculate the magnitude of the horizontal component force (F_X). Show your math work and record your answer in the space provided.

F_X = ________________lb

3. Calculate the magnitude of the vertical component force (F_Y). Show your math work and record your answer in the space provided.

F_Y = ________________lb

TIP SHEET

Vectors are often represented on a two-dimensional Cartesian grid with their tails at the intersection where $x = 0$ and $y = 0$. This has already been shown in many figures in this chapter, including Figure 12-6. In all these instances, however, all vectors were shown in Quadrant I (Figure 12-7). In Quadrant I, vectors (again, with their tails at $x = 0$ and $y = 0$), point up and to the right, having positive x and y values. However, if a vector were in Quadrant II, it would still point upward, having a positive y value, but it would point to the left, indicating a negative x value. If a vector were in Quadrant III, pointing down and to the left, both the x and y values would be negative. Finally, if a vector were in Quadrant IV, pointing down and to the right, its y value would be negative and its x value would be positive.

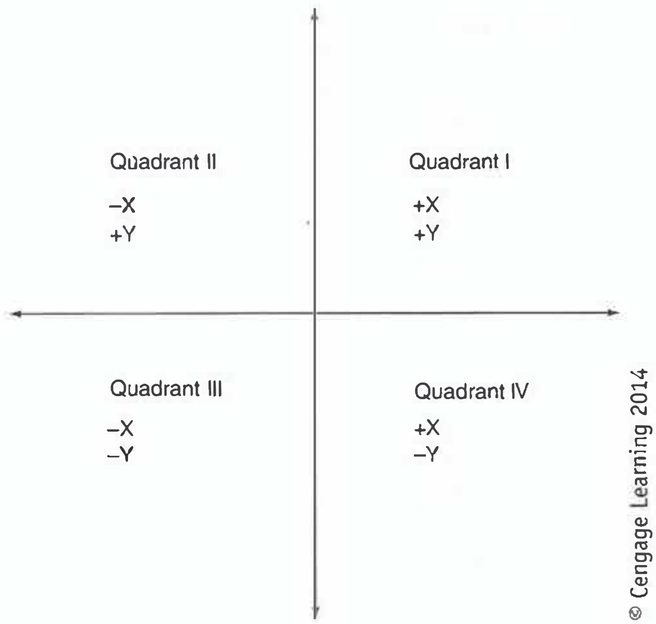

FIGURE 12-7 *The four quadrants of the Cartesian coordinate axes.*

The Cartesian grid provides a "sign convention" for assigning positive and negative values to vectors that are parallel to the x or y axis. A vector that is parallel to the x axis (e.g., F_x) that points to the right is given a positive sign, and one that points to the left is given a negative sign. Likewise, a vector that is parallel to the y axis (e.g., F_y) that points upward is given a positive sign, and one that points downward is given a negative sign. When adding component vectors together, their signs must be considered (Figures 12-8 and 12-9).

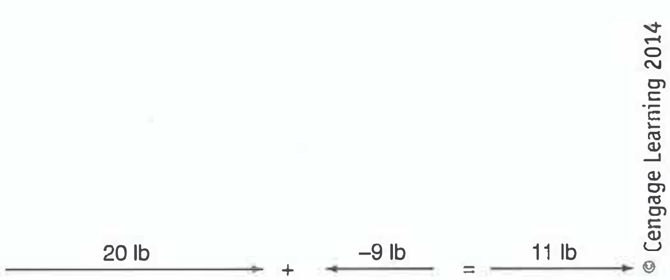

FIGURE 12-8 *Adding horizontal component vectors.*

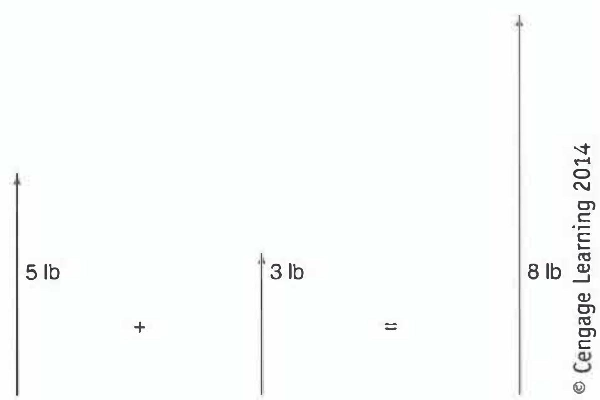

FIGURE 12-9 *Adding vertical component vectors.*

When multiple forces act upon an object at various angles, a good strategy to determine the resultant force acting on the object (in other words to add those vectors together) is to do the following:

1. Use trigonometry to break each vector down into component parts (F_x and F_y).
2. Use the sign convention described above to add all the forces in the x direction together to get the resultant force acting on the object in the x direction, F_{RX}.
3. Use the sign convention to add all the forces in the y direction together to get the resultant force acting on the object in the y direction, F_{RY}.
4. Use the Pythagorean theorem to determine the magnitude of the total force, F_R.
5. Use trigonometry to determine the direction (or angle) at which the total force acts.

Problem 12.6 Figure 12-10 is the vector representation of a 58-lb force (F_1) exerted at a downward angle of 30° relative to the horizontal axis and an 80-lb force (F_2) exerted at an upward angle of 60° relative to the horizontal axis.

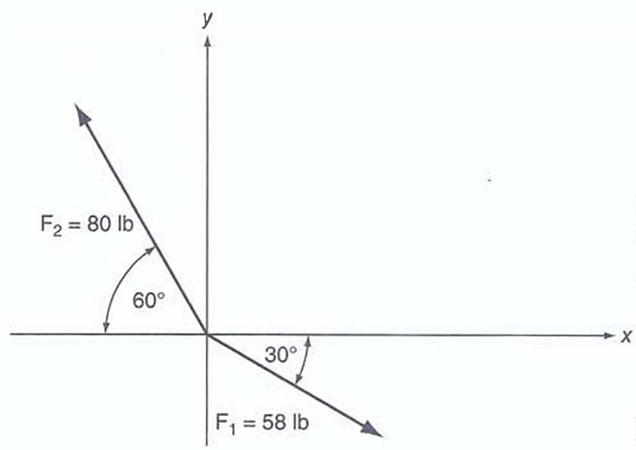

FIGURE 12-10 *Vector resolution—component method.*

1. Calculate the x and y components of force F_1, which applies a force down and to the right. Show your math work and record your answers in the space provided. Use appropriate signs in your answers.

F_{1X} = __________lb F_{1Y} = __________lb

2. Calculate the x and y components of force F_2, which applies a force up and to the left. Show your math work and record your answers in the space provided. Use appropriate signs in your answers.

F_{2X} = __________lb F_{2Y} = __________lb

3. Calculate the resultant force exerted in the x direction, F_{RX}, and in the y direction, F_{RY}. Show your math work and record your answers in the space provided. Use appropriate signs in your answers.

F_{RX} = ____________lb F_{RY} = ____________lb

4. Calculate the magnitude and direction of the resultant force, F_R. Show your math work and record your answers in the space provided.

F_R = ____________lb θ = ____________°

5. Using the values from Problem 12.5(3), sketch and label the resultant force vector F_R on Figure 12-10.

Problem 12.7 Figure 12-11 shows an eye hook assembly that is attached to a load and is supported by two separate cables. F_1 and F_2 represent the tension force in the cables of 150 lb and 80 lb, respectively.

6. Use the space provided to draw F_1 and F_2 on a Cartesian grid.

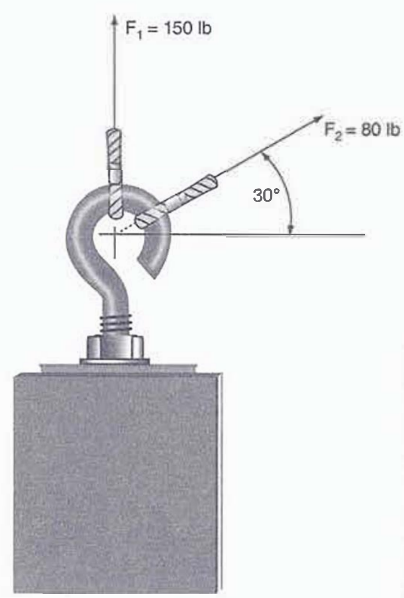

FIGURE 12-11 *Eye hook assembly.*

7. What are the x and y components of force F_1? Record your answers in the space provided. Use appropriate signs in your answers.

F_{1X} = ____________lb F_{1Y} = ____________lb

8. Calculate the x and y components of force F_2. Show your math work and record your answers in the space provided. Use appropriate signs in your answers.

F_{2X} = ____________lb F_{2Y} = ____________lb

9. Calculate the resultant force that the cables exert on the eye hook in the x direction, F_{RX}, and in the y direction, F_{RY}. Show your math work and record your answers in the space provided. Use appropriate signs in your answers.

F_{RX} = ____________lb F_{RY} = ____________lb

10. Calculate the magnitude and direction of the resultant force. Show your math work and record your answers in the space provided.

F_R = ____________lb θ = ____________°