Calculating Truss Forces

Method of Joints
Forces

Compression

A body being squeezed

Tension

A body being stretched
A truss is composed of slender members joined together at their end points.

– They are usually joined by welds or gusset plates.
A simple truss is composed of triangles, which will retain their shape even when removed from supports.
Pinned and Roller Supports

A **pinned** support can support a structure in two dimensions.

A **roller** support can support a structure in only one dimension.
Solving Truss Forces

Assumptions:

All members are perfectly straight.

All loads are applied at the joints.

All joints are pinned and frictionless.

Each member has no weight.

Members can only experience tension or compression forces.

What risks might these assumptions pose if we were designing an actual bridge?
Static Determinacy

A statically determinate structure is one that can be mathematically solved.

\[ 2J = M + R \]

where:

- \( J \) = Number of Joints
- \( M \) = Number of Members
- \( R \) = Number of Reactions
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**Try It**

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Try It

$$2J = M + R$$
Each side of the main street bridge in Brockport, NY has 19 joints, 35 members, and three reaction forces (pin and roller), making it a statically determinate truss.

\[2J = M + R\]

\[2(19) = 35 + 3\]

\[38 = 38\]
Equilibrium Equations

\[ \sum M = 0 \]

The sum of the moments about a given point is zero.
The sum of the forces in the x-direction is zero.

Do you remember the Cartesian coordinate system? A vector that acts to the right is positive, and a vector that acts to the left is negative.
Equilibrium Equations

$$\sum F_y = 0$$

The sum of the forces in the y-direction is zero.

A vector that acts up is positive, and a vector that acts down is negative.
Using Moments to Find $R_{Cy}$

A force that causes a clockwise moment is a negative moment.

A force that causes a counterclockwise moment is positive moment.

$F_D$ contributes a negative moment because it causes a clockwise moment about A.

$R_{Cy}$ contributes a positive moment because it causes a counterclockwise moment around A.

\[ \Sigma M_A = 0 \]
\[ -F_D(3.0 \text{ ft}) + R_{Cy}(10.0 \text{ ft}) = 0 \]
\[ -500 \text{ lb} \cdot (3.0 \text{ ft}) + R_{Cy}(10.0 \text{ ft}) = 0 \]
\[ -1500 \text{ lb} \cdot \text{ ft} + R_{Cy}(10.0 \text{ ft}) = 0 \]
\[ R_{Cy}(10.0 \text{ ft}) = 1500 \text{ lb} \cdot \text{ ft} \]
\[ R_{Cy} = 150 \text{ lb} \]
We know two out of the three forces acting in the y-direction. By simply summing those forces together, we can find the unknown reaction at point A.

Please note that a negative sign is in front of $F_D$ because the drawing shows the force as down.

\[ \Sigma F_y = 0 \]
\[ -F_D + R_{Cy} + R_{Ay} = 0 \]
\[ -500.\text{lb} + 150.00\text{lb} + R_{Ay} = 0 \]
\[ -350.\text{lb} + R_{Ay} = 0 \]

\[ R_{Ay} = 350.\text{lb} \]
Sum the x Forces to Find $A_x$

Because joint $A$ is pinned, it is capable of reacting to a force applied in the x-direction.

However, since the only load applied to this truss ($F_D$) has no x-component, $R_{Ax}$ must be zero.

\[ \sum F_x = 0 \]

\[ R_{Ax} = 0 \]
Method of Joints

Use cosine and sine to determine x and y vector components.

Assume all members to be in tension. A positive answer will mean the member is in tension, and a negative number will mean the member is in compression.

As forces are solved, update free body diagrams. Use correct magnitude and sense for subsequent joint free body diagrams.
Method of Joints

Truss Dimensions

\[ \theta_1 \]
\[ \theta_2 \]

\[ R_{Ax} \]
\[ R_{Ay} \]
\[ R_{Cy} \]

3.0 ft \hspace{50pt} 7.0 ft \hspace{50pt} 4.0 ft

500lb
Method of Joints

Using Truss Dimensions to Find Angles

$$\tan \theta_1 = \frac{\text{opp}}{\text{adj}}$$

$$\tan \theta_1 = \frac{4.0 \text{ ft}}{3.0 \text{ ft}}$$

$$\theta_1 = \tan^{-1} \frac{4.0}{3.0}$$

$$\theta_1 = 53.130^\circ$$
Method of Joints

Using Truss Dimensions to Find Angles

\[ \tan \theta_1 = \frac{\text{opp}}{\text{adj}} \]

\[ \tan \theta_1 = \frac{4.0 \text{ ft}}{7.0 \text{ ft}} \]

\[ \theta_1 = \tan^{-1} \frac{4.0}{7.0} \]

\[ \theta_1 = 29.745^\circ \]
Every member is assumed to be in tension. A positive answer indicates the member is in tension, and a negative answer indicates the member is in compression.
Where to Begin

Choose the joint that has the least number of unknowns.

Reaction forces at joints A and C are both good choices to begin our calculations.
Method of Joints

\[ \sum F_y = 0 \]
\[ R_{Ay} + AB_y = 0 \]
\[ 350 \text{lb} + AB \sin 53.130^\circ = 0 \]
\[ AB \sin 53.130^\circ = -350 \text{lb} \]
\[ AB = \frac{-350 \text{lb}}{\sin 53.130^\circ} \]
\[ AB = -438 \text{ lb} \quad \text{COMPRESSION} \]
Method of Joints

Update the all force diagrams based on AB being under compression.

\[ R_{Ax} = 0 \]
\[ R_{Ay} = 350\text{lb} \]
\[ 500\text{lb} \]
\[ R_{Cy} = 150\text{lb} \]
Method of Joints

\[ \Sigma F_x = 0 \]

\[-AB_x + AD = 0 \]

\[-437.50 \text{ lb} \cos 53.130^\circ + AD = 0 \]

\[ AD = 437.50 \text{ lb} \cos 53.130^\circ \]

\[ AD = 262.50 \text{ lb} \]
Method of Joints

\[
\sum F_Y = 0
\]
\[
R_{Cy} + BC \sin 29.745^\circ = 0
\]
\[
150 \, lb + BC \sin 29.745^\circ = 0
\]
\[
BC \sin 29.745^\circ = -150 \, lb
\]
\[
BC = \frac{-150 \, lb}{\sin 29.745^\circ}
\]
\[
BC = -302 \, lb \quad \text{COMPRESSION}
\]
Method of Joints

Update the all force diagrams based on BC being under compression.

RAy = 350lb
R_{Ay} = 350lb

R_{Ax} = 0

RCy = 150lb

RAx = 0
Method of Joints

\[ \Sigma F_x = 0 \]

\[ BC_x - CD = 0 \]

\[ 302.33 \text{ lb} \cos 29.745^\circ - CD = 0 \]

\[ CD = 302.33 \text{ lb} \cos 29.745^\circ \]

\[ CD = 262.50 \text{ lb} \]

TENSION
Method of Joints

\[ \Sigma F_Y = 0 \]
\[ BD - F_D = 0 \]
\[ BD - 500\text{lb} = 0 \]
\[ BD = 500\text{lb} \quad \text{TENSION} \]